ELSEVIER

Applied Surface Science 102 (1996) 308–313

applied surface science

Selection rules for light scattering by folded acoustic phonons in low-index Si-based superlattices

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Abstract

We consider the propagation of acoustic waves in Si-based heterojunctions (HJs), quantum wells (QWs) and superlattices (SLs) grown in arbitrary directions, and present a general formalism for obtaining wave velocities, selection rules, and efficiency of Raman scattering (RS) and Brillouin scattering (BS) by folded acoustic-phonons. Results based on nine different directions for the phonon wavevector are tabulated.

1. Introduction

The optical properties of SLs and quantum wells, grown on substrates with orientations other than (100), have been studied extensively in III–V compounds, especially in GaAs/AlAs systems (e.g. [012], [331], [311], Ref. [1]). The increased anisotropy introduces possibilities for new physical phenomena and device applications. The recent construction of low-index facets in Si, such as (111) [2], (113), (114) [3], leaves much to be expected also for low-index Si-based systems [4]. In anticipation of growing interest in such systems grown along arbitrary directions, we consider the propagation of acoustic waves and present a general formalism for obtaining wave velocities, selection rules, and efficiencies for Raman and Brillouin scattering by folded acoustic phonons.

2. Velocities of acoustic waves in arbitrary directions

The propagation of acoustic waves in SLs is dictated by the propagation of acoustic waves in the parent bulk materials. The analytic formula relating the wave velocity in a given direction to the elastic constants of the bulk solid is obtained from the solution of the mode secular equation [5]

$$|\mathbf{c}_{\lambda\mu\nu\rho} s_\mu s_\rho - \rho v^2 \delta_{\lambda\nu}| = 0 \qquad (1)$$

where $\mathbf{c}_{\lambda\mu\nu\rho}$ is the component of the elastic stiffness tensor, ρ is the mass density of the medium, v the phase velocity, $\delta_{\lambda\nu}$ the Kronecker delta, and $\mathbf{s}$ the unit vector in the direction of propagation which is also the direction of the wavevector $\mathbf{q}$, i.e., $\mathbf{q} = q\mathbf{s}$. Eq. (1) is valid in any system of axes $x'_1 x'_2 x'_3$, as well as in the system $x_1 x_2 x_3$ of crystallographic axes $\langle 100 \rangle$, provided the rotated components of $\mathbf{c}_{\lambda\mu\nu\rho}$ are known. In order to solve (1) for arbitrary $\mathbf{s}$, we choose $x'_3 \parallel \mathbf{s}$; the axes x'_1, x'_2 are conveniently chosen to complete a right-handed orthonormal sys-

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PII S0169-4332(96)00070-0

tem with unit vectors $\hat{\mathbf{x}}'_1$, $\hat{\mathbf{x}}'_2$, $\hat{\mathbf{x}}'_3$. This is the **q-adapted system of axes**. It is assumed that the transformation matrix which takes $x_1 x_2 x_3$ to $x'_1 x'_2 x'_3$ is known in the form

$$\mathbf{R} = \begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix} \tag{2}$$

where l_λ, m_λ, n_λ ($\lambda = 1, 2, 3$) represent the direction cosines of x'_λ relative to x_1, x_2, x_3, respectively. From (2), the following symmetric fourth-rank tensor is easily constructed

$$T_{\lambda\mu\nu\rho} = l_\lambda l_\mu l_\nu l_\rho + m_\lambda m_\mu m_\nu m_\rho + n_\lambda n_\mu n_\nu n_\rho = T_{ij} = T_{ji} \tag{3}$$

where i and j are the suppressed indices for $(\lambda\mu)$ and $(\nu\rho)$ [6]. Then [7],

$$c'_{ij} = c'_{\lambda\mu\nu\rho} = c_{\lambda\mu\nu\rho} + c\left(T_{\lambda\mu\nu\rho} - T^0_{\lambda\mu\nu\rho}\right) \tag{4}$$

where $c = c_{11} - c_{12} - 2c_{44}$ is the anisotropy factor, and T^0_{ij} equals one for $i = j = 1, 2, 3$ and zero

Table 1
Rotated stiffness coefficients c'_{ij} of cubic crystals, according to Eq. (5). The rotation matrices (unnormalized) are shown in the left column. In all cases the third row of the matrix corresponds to the direction of **q**. The anisotropy factor is $c = c_{11} - c_{12} - 2c_{44}$

$\begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix}$	c'_{55}	c'_{44}	c'_{43}	c'_{33}
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	c_{44}	c_{44}	0	c_{11}
$\begin{pmatrix} 0 & 0 & 1 \\ 1 & \bar{1} & 0 \\ 1 & 1 & 0 \end{pmatrix}$	c_{44}	$c_{44} + c/2$	0	$c_{11} - c/2$
$\begin{pmatrix} 0 & 0 & 1 \\ 1 & \bar{2} & 0 \\ 2 & 1 & 0 \end{pmatrix}$	c_{44}	$c_{44} + 8c/25$	$6c/25$	$c_{11} - 8c/25$
$\begin{pmatrix} 0 & 0 & 1 \\ 1 & \bar{3} & 0 \\ 3 & 1 & 0 \end{pmatrix}$	c_{44}	$c_{44} + 9c/50$	$6c/25$	$c_{11} - 9c/50$
$\begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & \bar{2} \\ 1 & 1 & 1 \end{pmatrix}$	$c_{44} + c/3$	$c_{44} + c/3$	0	$c_{11} - 2c/3$
$\begin{pmatrix} 0 & 1 & \bar{1} \\ \bar{1} & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$	$c_{44} + c/6$	$c_{44} + c/3$	0	$c_{11} - c/2$
$\begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & \bar{4} \\ 2 & 2 & 1 \end{pmatrix}$	$c_{44} + 4c/9$	$c_{44} + 4c/27$	$\frac{4c}{27\sqrt{2}}$	$c_{11} - 16c/27$
$\begin{pmatrix} 0 & 1 & \bar{1} \\ \bar{2} & 3 & 3 \\ 3 & 1 & 1 \end{pmatrix}$	$c_{44} + c/11$	$c_{44} + 27c/121$	$-\frac{48c}{121\sqrt{2}}$	$c_{11} - 38c/121$
$\begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & \bar{6} \\ 3 & 3 & 1 \end{pmatrix}$	$c_{44} + 9c/19$	$c_{44} + 27c/361$	$\frac{48c}{361\sqrt{2}}$	$c_{11} - 198c/361$

otherwise. It is emphasized that in this form, Eq. (4) holds only in cubic classes, for fourth-rank tensors with three independent components (for other situations see [7]). Since $\hat{\mathbf{x}}'_3 = [l_3 m_3 n_3]$ is chosen along **s** we have $s'_1 = s'_2 = 0$, $s'_3 = 1$, and Eq. (1) becomes

$$\begin{vmatrix} c'_{55} - \rho v^2 & c'_{54} & c'_{53} \\ c'_{45} & c'_{44} - \rho v^2 & c'_{43} \\ c'_{35} & c'_{34} & c'_{33} - \rho v^2 \end{vmatrix} = 0 \qquad (5)$$

Diagonalization of Eq. (5) yields the three eigenvalues (wave velocities) and the corresponding eigenvectors (polarizations) relative to $x'_1 x'_2 x'_3$. The amount of computation required for diagonalizing (5) for low-index directions is substantially reduced in comparison with the situation in the unrotated system $x_1 x_2 x_3$.

The necessary values of c'_{ij} for nine directions of **q** are given in Table 1. All directions **q** considered here are of the form $[pq0]$, $[pqq]$ or $[ppq]$. In all these cases, the axes x'_1, x'_2 (i.e., the first and second row of each matrix, respectively) are such that $c'_{54} = c'_{53} = 0$, and for this reason they are not included in the table (for different choices of the two axes this may not be true, however). The eigenvalues (ρv^2) for these situations are obtained from Eq. (5) and are presented in Table 2 [8]; since $c'_{54} = c'_{53} = 0$, there is at least one purely transverse (TA) mode polarized along x'_1 according to Eq. (5); the remaining two modes are, in general, quasi-longitudinal (QLA) and quasi-transverse (QTA), i.e., their eigenvectors are

Table 2

Sound velocities in cubic crystals for different directions of **q** (first column, unnormalized). The unnormalized rotation matrices are as in the first column of Table 1. For TA and LA modes, the entries (c_{44}, etc.) are eigenvalues of the corresponding mode secular equation, and the superscript directions are the corresponding mode eigenvectors (polarizations). For quasi-LA and quasi-TA modes, designated by QLA/QTA, the superscript directions do not coincide with the eigenvectors; the entries designate the two ρv^2 eigenvalues, corresponding to $\pm D$. p and q stand for normalized direction cosines of $\mathbf{q}_{\parallel} \hat{x}'_3$

$\mathbf{q}_{\parallel}$	ρv^2
001	$\mathrm{TA}^{[100]}$: c_{44} $\mathrm{TA}^{[010]}$: c_{44} $\mathrm{LA}^{[001]}$: c_{11}
110	$\mathrm{TA}^{[001]}$: c_{44} $\mathrm{TA}^{[1\bar{1}0]}$: $(c_{11} - c_{12})/2$ $\mathrm{LA}^{[110]}$: $(c_{11} + c_{12} + 2c_{44})/2$
111	$\mathrm{TA}^{[1\bar{1}0]}$: $(c_{11} - c_{12} + c_{44})/3$ $\mathrm{TA}^{[11\bar{2}]}$: $(c_{11} - c_{12} + c_{44})/3$ $\mathrm{LA}^{[111]}$: $(c_{11} + 2c_{12} + 4c_{44})/3$
211	$\mathrm{TA}^{[01\bar{1}]}$: $(c_{11} - c_{12} + 4c_{44})/6$
	$\mathrm{QLA}^{[211]}/\mathrm{QTA}^{[\bar{1}11]}$: $(5c_{11} + c_{12} + 8c_{44} \pm D)/12$ $D = \sqrt{(-3c_{11} + c_{12} + 4c_{44})^2 + 32(c_{12} + c_{44})^2}$
210	$\mathrm{TA}^{[001]}$: c_{44}
	$\mathrm{QLA}^{[210]}/\mathrm{QTA}^{[1\bar{2}0]}$: $(5c_{11} + 5c_{44} \pm D)/10$ $D = \sqrt{9(c_{11} - c_{44})^2 + 16(c_{12} + c_{44})^2}$
221	$\mathrm{TA}^{[1\bar{1}0]}$: $(4c_{11} - 4c_{12} + c_{44})/9$
	$\mathrm{QLA}^{[221]}/\mathrm{QTA}^{[11\bar{4}]}$: $(5c_{11} + 4c_{12} + 17c_{44} \pm D)/18$ $D = \sqrt{(3c_{11} + 4c_{12} + c_{44})^2 + 32(c_{12} + c_{44})^2}$
310	$\mathrm{TA}^{[001]}$: c_{44}
	$\mathrm{QLA}^{[310]}/\mathrm{QTA}^{[1\bar{3}0]}$: $(5c_{11} + 5c_{44} \pm D)/10$ $D = \sqrt{16(c_{11} - c_{44})^2 + 9(c_{12} + c_{44})^2}$
311	$\mathrm{TA}^{[01\bar{1}]}$: $(c_{11} - c_{12} + 9c_{44})/11$
	$\mathrm{QLA}^{[311]}/\mathrm{QTA}^{[\bar{2}33]}$: $(10c_{11} + c_{12} + 13c_{44} \pm D)/22$ $D = \sqrt{(8c_{11} - c_{12} - 9c_{44})^2 + 72(c_{12} + c_{44})^2}$
331	$\mathrm{TA}^{[1\bar{1}0]}$: $(9c_{11} - 9c_{12} + c_{44})/19$
	$\mathrm{QLA}^{[331]}/\mathrm{QTA}^{[11\bar{6}]}$: $(10c_{11} + 9c_{12} + 37c_{44} \pm D)/38$ $D = \sqrt{(8c_{11} + 9c_{12} + c_{44})^2 + 72(c_{12} + c_{44})^2}$
$pq0$	$\mathrm{TA}^{[110]}$: c_{44}
	QLA/QTA: $(c_{11} + c_{44} \pm D)/2$ $D = \sqrt{(p^2 - q^2)^2(c_{11} - c_{44})^2 + 4p^2q^2(c_{12} + c_{44})^2}$
pqq	$\mathrm{TA}^{[01\bar{1}]}$: $c_{44} + (c_{11} - c_{12} - 2c_{44})q^2$
	$\mathrm{QLA}^{[pqq]}/\mathrm{QTA}^{[\overline{2q}pp]}$: $[(p^2 + q^2)c_{11} + q^2c_{12} + (2 - p^2)c_{44} \pm D]/2$
	$D = \sqrt{[(p^2 - q^2)c_{11} - q^2c_{12} - p^2c_{44}]^2 + 8p^2q^2(c_{12} + c_{44})^2}$

The last two rows refer to the general cases of $[pq0]$ and $[pqq]$. Contrary to all other cases, these two unit vectors are normalized, i.e., $\sqrt{p^2 + q^2} = 1$, and $\sqrt{p^2 + 2q^2} = 1$, respectively.

somewhere between x'_2 and x'_3. In the event that $c'_{43} = 0$, the remaining two modes are polarized along x'_2 and x'_3, and are TA and LA, respectively. This is the case of high-symmetry directions.

Repeating the above procedure twice, once for each constituent bulk material 1 and 2, yields v_1 and v_2 and the corresponding eigenvectors. The velocity of propagating folded acoustic modes in SLs (layer thicknesses d_1, d_2, total thickness d) is equal to $v_1 v_2 d/(v_1 d_2 + v_2 d_1)$ for the same direction of $\mathbf{q}$. The eigenvectors are the same as for the bulk materials.

3. Selection rules

In the elastic approximation, the selection rules for RS by folded acoustic phonons in SLs are those of BS from the constituent crystals in bulk form. The latter are also applicable to HJs and QWs. In BS, the long wavelength acoustic modes produce strains $e_{\nu\rho}(\mathbf{r},t)$ which, in turn, modulate the dielectric constant through the elasto-optical effect. Specifically, an acoustic mode with frequency ω and wavevector $\mathbf{q}$ induces a time dependent change $\delta\epsilon(\mathbf{r},t)$ in the dielectric tensor which, for small strains, can be written as [6]

$$-\frac{1}{\epsilon^2}\delta\epsilon_{\lambda\mu}(\mathbf{r},t) = p_{\lambda\mu\nu\rho}e_{\nu\rho}(\mathbf{r},t) \tag{6}$$

where $p_{\lambda\mu\nu\rho}$ are the Pockels elasto-optical coefficients. The strains $e_{\nu\rho}(\mathbf{r},t)$ are related to the atomic displacements $U_\nu(\mathbf{r},t)$ by

$$e_{\nu\rho}(\mathbf{r},t) = \tfrac{1}{2}\left[\frac{\partial U_\nu(\mathbf{r},t)}{\partial x_\rho} + \frac{\partial U_\rho(\mathbf{r},t)}{\partial x_\nu}\right] \tag{7}$$

All quantities in (6) and (7), e.g. p_{ijkl} and e_{kl}, must be transformed to the $x'_1x'_2x'_3$ system. The Pockels coefficients for the III–V, II–VI, and diamond-type crystals transform exactly as the elastic stiffness coefficients c'_{ij} [set p for c everywhere in Eq. (4)]. On the other hand, $\mathbf{U}'(\mathbf{r}',t)$ can be written in the form of a plane wave

$$\begin{aligned}\mathbf{U}'(\mathbf{r}',t) &= \mathbf{U}_0\exp\mathrm{i}(\mathbf{q}\cdot\mathbf{r}' - \omega t)\\ &= \mathbf{U}'(\mathbf{r}')\exp(-\mathrm{i}\omega t)\end{aligned} \tag{8}$$

Since $\hat{x}'_3 \parallel \mathbf{q}$, namely $\mathbf{q}\cdot\mathbf{r}' = qx'_3$, the components of $\mathbf{U}'(\mathbf{r}')$ become $u'_\lambda(\mathbf{r}') = u_{0\lambda}\exp(\mathrm{i}qx'_3)$. Then, from Eq. (7) and (8) we find the strain tensor in the primed system

$$\mathbf{e}' = \mathrm{i}q\begin{pmatrix} 0 & 0 & u'_1 \\ 0 & 0 & u'_2 \\ u'_1 & u'_2 & u'_3 \end{pmatrix} \tag{9}$$

(Factors of 2 are necessary for the conversion $2e_{\nu\rho} = e_j$ when $\nu \neq \rho$, [6].)

The BS intensity is proportional to $|\delta\epsilon_{\lambda\mu}|^2$, in the elastic continuum model. The scattering selection rules are obtained from the double product of $\delta\epsilon$ with the incident ($\mathbf{e}^{\mathrm{i}}$) and scattered ($\mathbf{e}^{\mathrm{s}}$) light polarization unit vectors. The scattering efficiency for a particular acoustic mode is proportional to $|e^{\mathrm{s}}_\lambda \delta\epsilon_{\lambda\mu} e^{\mathrm{i}}_\mu|^2$. In view of Eq. (9), the summation over ν, ρ in (6) leads to

$$\delta\epsilon \sim \begin{pmatrix} p'_{15} & p'_{65} & p'_{55} \\ & p'_{25} & p'_{45} \\ & & p'_{35} \end{pmatrix} u'_1 + \begin{pmatrix} p'_{14} & p'_{64} & p'_{54} \\ & p'_{24} & p'_{44} \\ & & p'_{34} \end{pmatrix} u'_2 + \begin{pmatrix} p'_{13} & p'_{63} & p'_{53} \\ & p'_{23} & p'_{43} \\ & & p'_{33} \end{pmatrix} u'_3 \tag{10}$$

These matrices are analogous to the usual Raman matrices for triply-degenerate Raman-and IR-active optical phonons in the same class of materials; they refer to $x'_1x'_2x'_3$, and so do the displacements u'_1, u'_2, u'_3, with $\hat{x}'_3 \parallel \mathbf{q}$. Furthermore, the full symmetry of T_{ij}, Eq. (3), requires that $T_{65} = T_{14}$, $T_{64} = T_{25}$, $T_{63} = T_{54} = T_{45}$, $T_{53} = T_{35}$ and $T_{43} = T_{34}$ with analogous equalities for p''s, i.e., $p'_{65} = p'_{14}$, etc. Thus, the number of p'_{ij} components are reduced to 12. Written in the same way as the Raman matrices are usually written, Eq. (10) becomes

$$\underset{x'_1}{\begin{pmatrix} p'_{15} & p'_{14} & p'_{55} \\ & p'_{25} & p'_{45} \\ & & p'_{35} \end{pmatrix}}, \quad \underset{x'_2}{\begin{pmatrix} p'_{14} & p'_{25} & p'_{45} \\ & p'_{24} & p'_{44} \\ & & p'_{34} \end{pmatrix}}, \quad \underset{x'_3}{\begin{pmatrix} p'_{13} & p'_{45} & p'_{35} \\ & p'_{23} & p'_{34} \\ & & p'_{33} \end{pmatrix}}. \tag{11}$$

The 12 components can be further reduced after the scattering geometry is fixed. As an example, we treat the selection rules for backscattering geometry along x'_3 for the most general orientation of x'_3. Conservation of momentum imposes that $\mathbf{q} \parallel \hat{x}'_3$. Therefore, the matrices of Eq. (11) can be used and the three possible scattering configurations yield, according to Eq. (11),

$$x'_3(x'_1 x'_1)\bar{x}'_3 \therefore p'_{15}(\parallel x'_1) + p'_{14}(\parallel x'_2) + p'_{13}(\parallel x'_3)$$
$$x'_3(x'_1 x'_2)\bar{x}'_3 \therefore p'_{14}(\parallel x'_1) + p'_{25}(\parallel x'_2) + p'_{45}(\parallel x'_3)$$
$$x'_3(x'_2 x'_2)\bar{x}'_3 \therefore p'_{25}(\parallel x'_1) + p'_{24}(\parallel x'_2) + p'_{23}(\parallel x'_3)$$

The 12 components are reduced to 7, and the interpretation of Eq. (12) is the same as for RS: for parallel light polarizations along x'_1 (first row), three acoustic modes, polarized along x'_1, x'_2, x'_3, are allowed by symmetry. Their intensities are $\sim |p'_{15}|^2$, $|p'_{14}|^2$ and $|p'_{13}|^2$, respectively. The interpretation of the remaining selection rules in Eq. (12) is similar.

The results described by Eq. (12) are now applied to the nine orientations of Tables 1 and 2, with $\mathbf{q} \parallel x'_3$. For all these cases, $p'_{15} = p'_{25} = p'_{35} = p'_{45} = 0$ and the 7 components of Eq. (12) are reduced to 4. These are computed and listed in Table 3. The

Table 3
Rotated elasto-optical coefficients p'_{ij} of diamond- and zincblende-type crystals according to Eq. (13) and the rotation matrices (unnormalized) shown in the left column. In all cases, the third row of the matrix corresponds to the direction of $\mathbf{q}$. Additional coefficients are required for applying the selection rules for the most general backscattering configuration, Eq. (12). These can be obtained from Table 1 simply by changing c_{ij} to p_{ij}. Here $p = p_{11} - p_{12} - 2p_{44}$

$\begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix}$	p'_{13}	p'_{14}	p'_{23}	p'_{24}
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	p_{12}	0	p_{12}	0
$\begin{pmatrix} 0 & 0 & 1 \\ 1 & \bar{1} & 0 \\ 1 & 1 & 0 \end{pmatrix}$	p_{12}	0	$p_{12} + p/2$	0
$\begin{pmatrix} 0 & 0 & 1 \\ 1 & \bar{2} & 0 \\ 2 & 1 & 0 \end{pmatrix}$	p_{12}	0	$p_{12} + 8p/25$	$-6p/25$
$\begin{pmatrix} 0 & 0 & 1 \\ 1 & \bar{3} & 0 \\ 3 & 1 & 0 \end{pmatrix}$	p_{12}	0	$p_{12} + 9p/50$	$-6p/25$
$\begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & \bar{2} \\ 1 & 1 & 1 \end{pmatrix}$	$p_{12} + p/3$	$p/(3\sqrt{2})$	$p_{12} + p/3$	$-p/(3\sqrt{2})$
$\begin{pmatrix} 0 & 1 & \bar{1} \\ \bar{1} & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$	$p_{12} + p/6$	$p/(3\sqrt{2})$	$p_{12} + p/3$	0
$\begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & \bar{4} \\ 2 & 2 & 1 \end{pmatrix}$	$p_{12} + 4p/9$	$2p/(9\sqrt{2})$	$p_{12} + 4p/27$	$-10p/(27\sqrt{2})$
$\begin{pmatrix} 0 & 1 & \bar{1} \\ \bar{2} & 3 & 3 \\ 3 & 1 & 1 \end{pmatrix}$	$p_{12} + p/11$	$3p/(11\sqrt{2})$	$p_{12} + 27p/121$	$15p/(121\sqrt{2})$
$\begin{pmatrix} 1 & \bar{1} & 0 \\ 1 & 1 & \bar{6} \\ 3 & 3 & 1 \end{pmatrix}$	$p_{12} + 9p/19$	$3p/(19\sqrt{2})$	$p_{12} + 27p/361$	$-105p/(361\sqrt{2})$

selection rules of Eq. (12) for backscattering along x'_3 in these nine cases finally become:

$$x'_3(x'_1 x'_1)\bar{x}'_3 \therefore p'_{14}(\| x'_2) + p'_{13}(\| x'_3)$$

$$x'_3(x'_1 x'_2)\bar{x}'_3 \therefore p'_{14}(\| x'_1)$$

$$x'_3(x'_2 x'_2)\bar{x}'_3 \therefore p'_{24}(\| x'_2) + p'_{23}(\| x'_3).$$

The selection rules based on Eq. (13) and Table 3 are in agreement with those in the literature for the high symmetry directions [100], [110], [111] ([8,9]) and [210] ([10]). In contrast, for some of the lower symmetry directions we have found some errors in published papers. In the case of (311) orientation, Ref. [11], a QTA mode was considered, polarized along the direction [322] instead of the direction [$\bar{2}$33] taken here; that produced a wrong combination of elasto-optical coefficients in the Brillouin tensor for this mode. A similar error was found in the case of [211]-oriented samples [12], where a QLA mode was considered with polarization along [122] instead of [211].

A final comment is necessary here. Eq. (13) is valid for BS in each of the two constituent bulk materials and the scattering efficiencies are proportional to the corresponding factor $|p'_{ij}|^2$. In SLs, the same selection rules described by Eqs. (10) to (13) are valid, but the efficiencies now are proportional to the factor $|\Delta p'_{ij}|^2$, where $\Delta p'_{ij}$ is the difference between the p'_{ij} values of materials 1 and 2 (Ref. [13]).

4. Conclusion

In conclusion, the propagation of acoustic waves in cubic crystals and the selection rules of light scattering by folded acoustic phonons were analyzed. A convenient mathematical formalism was employed for handling the fourth-rank tensor transformations in the **q**-adapted system of axes. In this way, the lengthy computations were shortcut and the results were put in forms which are easier to physically interpret. This procedure was applied in detail to backscattering geometry along nine different SL growth directions. The same procedure is applicable to any other orientation of the acoustic mode wavevector **q**, e.g. perpendicular to the superlattice direction of growth.

Acknowledgements

Supported by NTUA and Human Capital and Mobility Program NEXUSTASK.

References

[1] Z.V. Popovic, E. Richter, J. Spitzer, M. Cardona, J.A. Shields, R. Nötzel and K. Ploog, Phys. Rev. B 49 (1994) 7577.
[2] H. Hibino, Y. Homma and T. Ogino, Phys. Rev. B 51 (1995) 7753.
[3] S. Song and S.G.J. Mochrie, Phys. Rev. B 51 (1995) 10068.
[4] R. Schorer, G. Abstreiter, S. De Gironcoli, E. Molinari, H. Kibbel and H. Presting, Phys. Rev. B 51 (1995) 5406; A.B. Talochkin, V.A. Markov, I.G. Neizvestnyt, O.P. Pchelyakov, M.P. Sinyukov and S.I. Stenin, Pis'ma Zh. Eksp. Teor. Fiz. 50 (1989) 21.
[5] J. Sapriel, Acousto-Optics (J. Wiley, New York, 1979) p. 18.
[6] J.F. Nye, Physical Properties of Crystals (Oxford University Press, London, 1964).
[7] E. Anastassakis and E. Liarokapis, Phys. Stat. Sol. (b) 149 (1988) K1.
[8] Z.V. Popovic, M. Cardona, E. Richter, D. Strauch, L. Tapfer and K. Ploog, Phys. Rev. B40 (1989) 3040.
[9] Z.V. Popovic, M. Cardona, E. Richter, D. Strauch, L. Tapfer and K. Ploog, Phys. Rev. B41 (1990) 5904.
[10] Z.V. Popovic, J.H. Trodhal, M. Cardona, E. Richter, D. Strauch and K. Ploog, Phys. Rev. B40 (1989) 202.
[11] Z.V. Popovic, J. Spitzer, T. Ruf, M. Cardona, R. Nötzel and K. Ploog, Phys. Rev. B48 (1993) 659.
[12] Z.V. Popovic, E. Richter, J. Spitzer, M. Cardona, R. Nötzel and K. Ploog, Superlatt. Microstruc. 14 (1993) 173.
[13] C. Colvard, T.A. Gant, M.V. Klein, R. Merlin, R. Fischer, H. Morkoc and A.C. Gossard, Phys. Rev. B 31 (1985) 2080.